YITP-SB-00-02
hep-th/0002028

On the nonlinear KK reductions on spheres of supergravity theories

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Abstract

We address some issues related to the construction of general Kaluza-Klein (KK) ansätze for the compactification of a supergravity (sugra) theory on a sphere S_m. We first reproduce various ansätze for compactification to 7d from the ansatz for the full nonlinear KK reduction of 11d sugra on $AdS_7 \times S_4$. As a side result, we obtain a lagrangean formulation of 7d $\mathcal{N}=2$ gauged sugra, which so far had only a on-shell formulation, through field equations and constraints. The $AdS_7 \times S_4$ ansatz generalizes therefore all previous sphere compactifications to 7d. Then we consider the case when the scalars in the lower dimensional theory are in a coset $Sl(m+1)/SO(m+1)$, and we keep the maximal gauge group $SO(m+1)$. The 11-dimensional sugra truncated on S_4 fits precisely the case under consideration, and serves as a model for our construction. We find that the metric ansatz has a universal expression, with the internal space deformed by the scalar fluctuations to a conformally rescaled ellipsoid. We also find the ansatz for the dependence of the antisymmetric tensor on the scalars. We comment on the fermionic ansatz, which will contain a matrix U interpolating between the spinorial $SO(m+1)$ indices of the spherical harmonics and the R-symmetry indices of the fermionic fields in the lower dimensional sugra theory. We derive general conditions which the matrix U has to satisfy and we give a formula for the vielbein in terms of U. As an application of our methods we obtain the full ansatz for the metric and vielbein for 10d sugra on $AdS_5 \times S_5$ (with no restriction on any fields).

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1 Introduction

The problem of Kaluza-Klein (KK) reduction of sugras on spheres is a very difficult one. In general, one obtains a gauged sugra after KK reduction, and finding the nonlinear ansatz which realizes the embedding of this gauged sugra into the original model is quite nontrivial. In the case of the reduction of 11d sugra on $AdS_4 \times S_7$, parts of the ansatz where found in [1, 2, 3, 4]. In particular, the ansatz for the antisymmetric tensor was not explicitly obtained (only in a highly implicit form, not suitable for calculations). The nonlinear ansatz for the $AdS_7 \times S_4$ reduction of 11d sugra was found by the present authors together with Peter van Nieuwenhuzen in [5, 6], and was the first to give a complete solution for the nonlinear KK ansatz for a sphere compactification. The ansatz for the compactification of 10d IIB sugra on $AdS_5 \times S_5$ is the most notable absence in this respect. Another problem which has to be addressed is the *consistency* of the truncation (KK reduction). In [4], the consistency of the KK reduction of an SU(8)-invariant version of the 11d sugra was given. In [6], we gave a proof of consistency of the reduction of the usual 11d sugra on $AdS_7 \times S_4$. In this paper, we will not have anything to say about the consistency of the ansätze we will propose, except when we take various further truncations of our $AdS_7 \times S_4$ ansatz to compare to other work in the literature.

The recent interest in sphere compactifications came from the AdS-CFT correspondence [7, 8, 9, 10, 11, 12]. A lot of work has been done in finding ansatze for further truncations of the models after the KK reduction on spheres. In [13, 14] it was considered the further truncation of the KK reduction of 11d and 10d IIB sugras on $AdS_4 \times S_7, AdS_7 \times S_4$ and $AdS_5 \times S_5$ to subsets of U(1) gauge fields and scalars. In [15] was studied the truncation of the KK reduction of 11d sugra on $AdS_7 \times S_4$ to the bosonic sector of an N=2 (SU(2)) gauged sugra. A set of only gravity and scalar fields was considered in [16], whereas [17] analyzed various compactifications giving rise to gravity plus scalar fields in a coset manifold. In [18] it was found an explicit ansatz for the embedding of the N=4 (SO(4)) gauged sugra in 11d (via S_7 compactification). In [19, 20] the KK reduction of 10d sugra on $S_3 \times S_3$ was analyzed, and in [21] the embedding of N=4 gauged sugra into 10d sugra.

In this paper we want to generalize some of the results obtained in [6], most notably for application to the $AdS_4 \times S_7$ and $AdS_5 \times S_5$ cases, and also to relate our results to the other works in the literature. We will explicitly show that the other ansätze for embedding of subsets of fields of the 7d N=4 gauged sugra into 11 dimensions can be obtained as particular cases of the ansatz in [6]. More precisely, we will recover: the ansatz involving the graviton and 4 scalars in [16], the ansatz involving the graviton, an SU(2) gauge field, a scalar and an antisymmetric tensor in [15], the ansatz for the S_3 compactification of 10 d sugra of [22] (which leads to $d = 7$ $\mathcal{N} = 2$ gauged sugra without topological mass term) and the ansatz involving a graviton, two abelian gauge fields and two scalars in [13]. The correspondence between our ansatz truncated to the fields of $d = 7$ $\mathcal{N} = 2$ gauged sugra [33] and the ansatz of Lü and Pope [15] is discussed in detail. To obtain agreement between the two ansatze (and between the field equations derived from the truncated action of the $\mathcal{N} = 4$ model and the field equations of $\mathcal{N} = 2$ gauged sugra), we need to make a redefinition of the three index potential $A_{(3)}$ (in the $\mathcal{N} = 2$ model) as a linear combination of the three index potential

$S_{(3)}$ (in the $\mathcal{N} = 4$ model) and the Chern Simons form $\omega_{(3)}$. We noted that there is a lagrangean formulation of $d = 7$ $\mathcal{N} = 2$ gauged sugra, obtained from the maximal 7d gauged sugra lagrangean by truncatation, which yields the set of field equations and constraints which so far had been used to describe the $\mathcal{N} = 2$ model. By taking a singular limit of the ansatz which yields $\mathcal{N} = 2$ $d = 7$ gauged sugra from $d = 11$ sugra with topological mass term [15, 6], we obtain an ansatz which yields $\mathcal{N} = 2$ $d = 7$ gauged sugra without topological mass term. Thus we confirm the consistency of the ansatz proposed by Chamseddine and Sabra [22] for the S_3 compactification of type I 10d sugra to 7d, who derived their ansatz only from the requirement to reproduce the action of gauged $\mathcal{N} = 2$ $d = 7$ sugra (which does not necessarily imply consistency). Since we obtain it as a singular limit from a consistent ansatz we implicitly prove the consistency of the truncation in [22].

The generalizations we are interested in are related to the ansatz for the embedding of the lower (d-) dimensional scalar fields into the higher dimension (D). We will see that when one compactifies on a sphere S_n, one has a scalar coset which is at least $Sl(n+1)/SO(n+1)$. We will mostly work with this coset, but for the $AdS_4 \times S_7$ and $AdS_5 \times S_5$ compactifications we will analyze also the case of the full coset. One would think that only the cases (d,n)= (4,7),(7,4) and (5,5) are of interest, and the metric and antisymmetric tensor ansatz for this case were found in [16], but actually the case of general d and n has its own interest. That is for instance because it was found useful in problems related to the Randall-Sundrum model [23, 24], for instance see [25]. For Randall-Sundrum type scenarios, one needs models of gravity interacting with scalar fields. Since finding a consistent realization of this scenario inside a susy theory is an (yet) unsolved problem (see [26, 27, 28]), it is worth studying general gravity + scalar systems. Moreover, [16] considered only n scalars, which is OK as long as no gauge fields couple to them. But we find the metric and antisymmetric tensor ansatz and show how to couple the gauge fields to the scalars. A completely new result is that we obtain the full metric ansatz for the KK reduction of 10d IIB sugra on $AdS_5 \times S_5$, which means that if one has a solution of 5d gauged sugra one can find the metric of the full 10d solution, and sometimes that is all one needs (for instance for the applications for the Randall-Sundrum type scenarios).

Yet another extension of the ansatz in [6] is related to the fermions. We see in general that there will be a scalar field-dependent matrix U which relates the fermions to the Killing spinors, as was done in [1] for the $AdS_4 \times S_7$ case and in [5, 6] for the $AdS_7 \times S_4$ case (the matrix U in the 4d case was introduced also in [29, 30]). We find an ansatz for the vielbein which depends on the matrix U and an equation which U has to satisfy. We analyze separately the $AdS_4 \times S_7$ and $AdS_5 \times S_5$ cases, and we find there the vielbein and the U equation for the theory with the full scalar coset (no restriction to $Sl(n+1)/SO(n+1)$).

The paper is organized as follows. In section 2 we derive from the ansatz [6] for the consistent KK reduction of $d = 11$ sugra to maximal $d = 7$ gauged sugra the other ansätze in the literature: [15, 16, 13, 22, 14]. for the KK reduction to smaller bosonic sectors of the maximal gauged sugra in $d = 7$. In section 3 we write the metric ansatz for the S_n compactification involving a scalar coset $Sl(n+1)/SO(n+1)$, and the dependence of the antisymmetric tensor field on the scalars. For the $AdS_4 \times S_7$ and the $AdS_5 \times S_5$ cases, we get the full metric ansatz (with no restrictions). In section 4

we say a few words about the fermionic ansatz, introduce the matrix U, and derive the vielbein and U equation. Again, in the $AdS_4 \times S_7$ and $AdS_5 \times S_5$ cases, we impose no restriction. We finish with discussions and conclusions. In the Appendix we give some useful things about the Killing spinors on spheres.

2 From d=11 sugra to d=7 sugra via S_4 compactification

The consistent embedding of seven dimensional supergravity theories in higher dimensions was thoroughly studied in the recent literature. We will first describe the case of S_4 truncation of the original d=11 supergravity [5, 6], and then obtain other embeddings as particular cases. When compactifying on S_4 we obtain $\mathcal{N}=4$ maximal gauged sugra in seven dimensions [31]. The scalars ${\Pi_A}^i(y)$ parameterize an $Sl(5,\mathbf{R})/\mathbf{SO(5)_c}$ coset manifold. The gauge fields $B_\alpha^{AB}(y)$ are in the adjoint of $SO(5)_g$, and the gravitini and spin 1/2 fields transform in the spinorial representation of the R-symmetry group $SO(5)_c$ and in the vector-spinor representation, respectively. As we shall see, this model admits consistent truncations to smaller field subsets: $\mathcal{N}=2$ gauged sugra, [†] or purely bosonic sectors such as the graviton and the whole set of scalars, or the graviton, two abelian gauged fields (in the Cartan subalgebra of $SO(5)$) and two scalars. Our ansatz for the metric factorizes into a rescaled 7 dimensional metric and a gauge invariant two form which depends on the scalar fields through the composite tensor $T^{AB}(y) = {\Pi^{-1}}_i{}^A {\Pi^{-1}}_i{}^B$.

$$
\begin{aligned}
ds_{11}^2(x,y) &= \Delta^{-2/5}(x,y)\left[g_{\alpha\beta}(y)dy^\alpha dy^\beta + \frac{1}{m^2}(DY)^A \frac{T^{-1}_{AB}(y)}{Y\cdot T(y)\cdot Y}(DY)^B\right] \quad (2.1)\\
DY^A &= dY^A + 2mB^{AB}(y)Y_B \\
&\qquad (2.2)
\end{aligned}
$$

The scale factor is

$$
\Delta^{-6/5}(x,y) = Y^A(x)T_{AB}(y)Y^B(x) \qquad (2.3)
$$

The full internal metric $g_{\mu\nu} = m^{-2}\Delta^{4/5}\partial_\mu Y^A T^{-1}_{AB}\partial_\nu Y^B$ describes a conformally rescaled ellipsoid: $g_{\mu\nu} = m^{-2}\Delta^{4/5}\partial_\mu Z^A \partial_\nu Z_A$ where the $Z^A(x)$'s are constrained to lie on the ellipsoid. Therefore the overall effect of all scalar fluctuations on the internal metric is to deform the background sphere $\mathring{g}_{\mu\nu} = m^{-2}\partial_\mu Y^A \partial_\nu Y^B \delta_{AB}$, $Y^A(x)Y^B(x)\delta_{AB} = 1$ into a conformally rescaled ellipsoid. Other types of fluctuations would correspond to 'massive' modes.

The 4-form field strength of the three index "photon" of the standard $d=11$ sugra is given by

$$
\begin{aligned}
\frac{\sqrt{2}}{3}F_{(4),11} &= \epsilon_{A_1\ldots A_5}\left[-\frac{1}{3m^3}(DY)^{A_1}\ldots(DY)^{A_4}\frac{(T\cdot Y)^{A_5}}{Y\cdot T\cdot Y}\right.\\
&\quad + \frac{4}{3m^3}(DY)^{A_1}(DY)^{A_2}(DY)^{A_3}D\left(\frac{(T\cdot Y)^{A_4}}{Y\cdot T\cdot Y}\right)Y^{A_5}
\end{aligned}
$$

[†] We are thankful to C.N.Pope H.Lü and A.Sadrzadeh for pointing to us this fact.

$$+\frac{2}{m^2}F_{(2)}^{A_1A_2}(DY)^{A_3}(DY)^{A_4}\frac{(T\cdot Y)^{A_5}}{Y\cdot T\cdot Y}+\frac{1}{m}F_{(2)}^{A_1A_2}F_{(2)}^{A_3A_4}Y^{A_5}\Bigg]$$
$$+d(S_{(3)B}Y^B) \tag{2.4}$$

where we define the Yang-Mills field strength to be $F_{(2)}^{AB}=2(dB_{(1)}+(B_{(1)}\cdot B_{(1)})^{AB}$ and the independent fluctuation form $S_{(3)B\alpha\beta\gamma}=-\frac{8i}{\sqrt{3}}S_{\alpha\beta\gamma,B}$ is real. (We also used the implicit convention $F_{(4),11}=F_{\Lambda\Pi\Sigma\Omega}dx^\Lambda\wedge dx^\Pi\wedge dx^\Sigma\wedge dx^\Omega$.)

Finally, in order to produce a first order field equation for the antisymmetric tensor field $S_{\alpha\beta\gamma}$ (self-duality in odd dimensions [32]) as well as for the consistency of the fermionic susy transformation laws we had to introduce in the $d=11$ sugra model an auxiliary field $\mathcal{B}_{MNPQ}$. The ansatz for the KK truncation of the 4-index auxiliary field has a single nonvanishing component

$$\frac{\mathcal{B}_{\alpha\beta\gamma\delta}}{\sqrt{g_{11}}}=\frac{i}{2\sqrt{3}}\epsilon_{\alpha\beta\gamma\delta\epsilon\eta\zeta}\frac{\delta S_7}{\delta S_{\epsilon\eta\zeta,A}}Y^A \tag{2.5}$$

where S_7 is the action of the 7-dimensional gauged sugra. The other models studied in the literature (in $d=7$) are special cases of this general ansatz.

- graviton and scalars in $Sl(5)/SO(5)$[16]

$$ds_{11}^2 = \tilde{\Delta}^{1/3}ds_7^2+\frac{1}{g^2}\tilde{\Delta}^{-2/3}\sum_i X_i^{-1}d\mu_i^2 \tag{2.6}$$

$$F_{(7),11} = g\sum_i(2X_i^2\mu_i^2-\tilde{\Delta}X_i)\epsilon_{(7)}-\frac{1}{2g}\sum_i X_i^{-1}\star dX_i\wedge d(\mu_i^2) \tag{2.7}$$

where the scalar fields satisfy the constraint $\prod_{i=1}^5 X_i=1$, and spherical harmonics μ_i are such that $\sum_{i=1}^5\mu_i^2=1$. The conformal factor $\tilde{\Delta}$ is defined as $\tilde{\Delta}=\sum_{i=1}^5 X_i\mu_i^2$. We easily recognize that the two sets of spherical harmonics μ_i and our Y^A are the same. Moreover, the scalar fields X_i are embedded in T_{AB} as

$$T_{AB}=diag(X_1,X_2,\dots,X_5) \tag{2.8}$$

Therefore, $\Delta^{-6/5}=\tilde{\Delta}$, and the metric ansatz given in (2.6) coincides with the ansatz (2.1) if we make the identification (2.8), and we set the gauge fields to zero. However, it is not immediately obvious that the ansatz for the antisymmetric tensor in (2.7) also coincides with (2.4), upon truncation. To check agreement between the two ansatze we compute the dual of $F_{(7),11}$:

$$\begin{aligned}F_{(4),11} &= \hat{\star}F_{(7),11}=\\ &= \frac{\sqrt{g_{11}}}{4!}\epsilon^{\alpha_1\dots\alpha_7}{}_{\mu\nu\rho\sigma}g\sum_i(2X_i^2\mu_i^2-\tilde{\Delta}X_i)\frac{\sqrt{g_7}}{7!}\epsilon_{\alpha_1\dots\alpha_7}dx^\mu dx^\nu dx^\rho dx^\sigma\\ &\quad -\frac{\sqrt{g_{11}}}{4!}\epsilon^{\alpha_1\dots\alpha_6\mu}{}_{\alpha_7\nu\rho\sigma}\sum_i X_i^{-1}\partial_\alpha X_i\partial_\mu(\mu_i^2)\frac{\sqrt{g_7}}{6!2g}\epsilon_{\alpha_1\dots\alpha_6}{}^{\alpha}dy^{\alpha_7}dx^\nu dx^\rho dx^\sigma\end{aligned} \tag{2.9}$$

where the notation $\hat{\star}$ means Hodge dual in the higher dimensional space.

Using that the inverse metric is

$$\begin{aligned} g_{11}^{\alpha\beta} &= g_7^{\alpha\beta}\tilde{\Delta}^{-1/3} \\ g_{11}^{\mu\nu} &= g^2\tilde{\Delta}^{-1/3}\left[\frac{1}{4}\sum_i(\overset{\circ}{\partial}{}^{\mu}(\mu_i^2)X_i)\sum_j(\overset{\circ}{\partial}{}^{\nu}(\mu_j^2)X_j) - \tilde{\Delta}(\sum_i \overset{\circ}{\partial}{}^{\mu}(\mu_i)X_i\overset{\circ}{\partial}{}^{\nu}(\mu_i))\right] \end{aligned} \tag{2.10}$$

where $\overset{\circ}{\partial}{}^{\mu} = \overset{\circ}{g}{}^{\mu\nu}\partial_\nu$ and $\overset{\circ}{g}{}^{\mu\nu}$ is the inverse background metric on S_4, and that $\sqrt{g_{11}} = \sqrt{g_4}\sqrt{g_7}\tilde{\Delta}^{7/6} = \tilde{\Delta}^{1/3}\sqrt{g_7}\sqrt{\overset{\circ}{g}_4}g^{-4}$ we get

$$\begin{aligned} F_{(4)} &= \epsilon_{\mu\nu\rho\sigma}\frac{\sqrt{\overset{\circ}{g}_4}}{4!}\tilde{\Delta}^{-2}g^{-3}\sum_i(2X_i^2\mu_i^2 - \tilde{\Delta}X_i)dx^\mu dx^\nu dx^\rho dx^\sigma \\ &\quad +\epsilon_{\mu'\nu\rho\sigma}\frac{\sqrt{\overset{\circ}{g}_4}}{4!}\frac{\tilde{\Delta}^{-2}\tilde{\Delta}^{1/3}}{2g^3}g^{\mu\mu'}\sum_i X_i^{-1}\partial_\alpha X_i\partial_\mu(\mu_i^2)dy^\alpha dx^\nu dx^\rho dx^\sigma \end{aligned} \tag{2.11}$$

Using further that $\partial_\mu\mu_i\partial_\nu\mu_i = \overset{\circ}{g}_{\mu\nu}$ and that [6]

$$\epsilon_{ABCDE}dY^A \ldots dY^D = \sqrt{\overset{\circ}{g}_4}\epsilon_{\mu\nu\rho\sigma}Y^E dx^\nu dx^\rho dx^\sigma \tag{2.12}$$

we can show that (2.11) is a particular case of (2.4).

- graviton, SU(2) gauge fields $A^i_{(1)}$, a three form $A_{(3),7}$ and one scalar X
The ansatz for the consistent bosonic truncation of $d = 11$ sugra to $\mathcal{N} = 2$ $d = 7$ gauged sugra [33] was given by H.Lü and C.N.Pope [15]

$$ds_{11}^2 = \tilde{\Delta}^{1/3}ds_7^2 + \frac{2}{g^2}X^3\tilde{\Delta}^{1/3}d\xi^2 + \frac{1}{2g^2}\tilde{\Delta}^{-2/3}X^{-1}\cos^2\xi\sum_i(\sigma^i - gA^i_{(1)})^2 \tag{2.13}$$

$$\begin{aligned} F_{(4),11} &= -\frac{1}{2\sqrt{2}g^3}(X^{-8}\sin^2\xi - 2X^2\cos^2\xi + 3X^{-3}\cos^2\xi - 4X^{-3}) \\ &\quad \tilde{\Delta}^{-2}\cos^3\xi d\xi\wedge\epsilon_{(3)} - \frac{5}{2\sqrt{2}g^3}\tilde{\Delta}^{-2}X^{-4}\sin\xi\cos^4\xi dX\wedge\epsilon_{(3)} + \sin\xi F_{(4),7} \\ &+ \frac{\sqrt{2}}{g}\cos\xi X^4 \star F_{(4),7}\wedge d\xi - \frac{1}{\sqrt{2}g^2}\cos\xi F^i_{(2)}\wedge d\xi\wedge h^i \\ &- \frac{1}{4\sqrt{2}g^2}X^{-4}\tilde{\Delta}^{-1}\sin\xi\cos^2\xi F^i_{(2)}\wedge h^k\wedge h^k\epsilon_{ijk} \end{aligned} \tag{2.14}$$

where the 4-sphere is described as a foliation of 3-spheres, parametrized by the Euler angles (θ, φ, ψ) with latitude coordinate ξ; σ^i are left invariant one forms on S_3: $d\overset{\circ}{s}{}^2_4 = d\xi^2 + 1/4\cos^2\xi\sigma^i\sigma^i$; $F^i_{(2)} = dA^i_{(1)} + g/2\epsilon_{ijk}A^j_{(1)}A^k_{(1)}$ are the Yang Mills field strengths with g the coupling constant and $h^i = \sigma^i - gA^i_{(1)}$ are the gauge invariant one forms. $\omega_{(3)}$ is a Chern-Simons form, $d\omega_{(3)} = F^i_{(2)}\wedge F^i_{(2)}$. As discussed in [6] the maximal gauge field group inherited from the 4-sphere, $SO(5)$, needs

to be broken first to $SO(4)$ by first setting $B^{A5}=0$ and further, from $SO(4)$ to one of its $SU(2)$ subgroups, by imposing an antiself-duality condition. Thus the embedding of the gauge fields is

$$B^{A5}=0 \quad ; \quad B^{\hat{\mu}\hat{\nu}}=-\frac{1}{2}\epsilon^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}B_{\hat{\rho}\hat{\sigma}}, \hat{\mu}=1,4 \quad ; \quad B^{i4}=-\frac{1}{2\sqrt{2}}A^{i}_{(1)} \tag{2.15}$$

The scalar field of $\mathcal{N}=2$ gauged sugra is embedded in the $Sl(5)/SO(5)$ coset of maximal $\mathcal{N}=4$ sugra as

$$T_{AB}=diag(X,X,X,X,X^{-4}) \tag{2.16}$$

The three index form of maximal $d=7$ sugra $S^{A}_{(3)}$ is also truncated to a singlet under $SU(2)$

$$S_{(3)}\equiv S^{5}_{(3)} \quad ; \quad 0=S^{\hat{\mu}}_{(3)}\ ,\ \hat{\mu}=1,4 \tag{2.17}$$

The correspondence between the Cartesian spherical harmonics Y^{A} and the new parametrization of the 4-sphere is

$$Y^{5}=\sin\xi \quad ; \quad Y^{\hat{\mu}}=\cos\xi\hat{Y}^{\hat{\mu}}\ ,\ \hat{\mu}=1,4 \tag{2.18}$$

where $\hat{Y}^{\hat{\mu}}$ are constrained to lie on a 3-sphere. Finally, the Cartesian coordinates $\hat{Y}$ are related to the Euler angles (θ,φ,ψ) by

$$\begin{aligned}\hat{Y}^{4}+i\hat{Y}^{3}&=\cos\frac{\theta}{2}\exp(\frac{i(\varphi+\psi)}{2})\\ \hat{Y}^{2}+i\hat{Y}^{1}&=\sin\frac{\theta}{2}\exp(\frac{i(\psi-\varphi)}{2})\end{aligned} \tag{2.19}$$

and by organizing them into an $SU(2)$ matrix

$$\mathcal{G}=\begin{pmatrix}\hat{Y}^{4}+i\hat{Y}^{3} & \hat{Y}^{2}+i\hat{Y}^{1}\\ -\hat{Y}^{2}+i\hat{Y}^{1} & \hat{Y}^{4}-i\hat{Y}^{3}\end{pmatrix}=\hat{Y}^{4}\mathbb{1}+i\tau^{k}\hat{Y}^{k} \tag{2.20}$$

we get the invariant $SU(2)$ one forms σ^{i} in terms of the Cartesian coordinates from

$$\sigma^{i}=\frac{1}{i}tr(\tau^{i}\mathcal{G}^{-1}d\mathcal{G})=2(\hat{Y}^{4}d\hat{Y}^{i}-\hat{Y}^{i}d\hat{Y}^{4}+\epsilon_{ijk}\hat{Y}^{j}d\hat{Y}^{k}) \tag{2.21}$$

Also, from (2.21) we get

$$d\sigma^{i}=4(-d\hat{Y}^{i}d\hat{Y}^{4}+\frac{1}{2}\epsilon_{ijk}d\hat{Y}^{j}d\hat{Y}^{k}) \tag{2.22}$$

Thus, the equivalence between the two metric ansatze is straightforward [6]. In (2.1) we substitute the truncated fields and the redefined spherical harmonics and we get

$$\begin{aligned}ds^{2}_{11} &= \Delta^{-2/5}(x,y)\left[g_{\alpha\beta}(y)dy^{\alpha}dy^{\beta}+\frac{1}{m^{2}}(DY)^{A}\frac{T^{-1}_{AB}(y)}{Y\cdot T(y)\cdot Y}(DY)^{B}\right]\\ &= \tilde{\Delta}^{1/3}ds^{2}_{7}+m^{-2}\tilde{\Delta}^{1/3}X^{3}d\xi^{2}+m^{-2}\tilde{\Delta}^{-2/3}\cos^{2}\xi X^{-1}\Big(d\hat{Y}^{\hat{\mu}}d\hat{Y}^{\hat{\mu}}\\ &+ 2m(B\cdot\hat{Y})^{\hat{\mu}}d\hat{Y}^{\hat{\mu}}+m^{2}(B\cdot\hat{Y})^{\hat{\mu}}(B\cdot\hat{Y})^{\hat{\mu}}\Big)\end{aligned} \tag{2.23}$$

Using the relation (2.21) and the anti-selfduality condition on the gauge fields, the 11-dimensional invariant line element becomes:

$$ds_{11}^2 = \tilde{\Delta}^{1/3} ds_7^2 + \frac{2}{g^2}\tilde{\Delta}^{1/3} X^3 d\xi^2 + \frac{1}{2g^2}\cos^2\xi X^{-1}(\sigma^i - gA^i_{(1)})(\sigma^i - gA^i_{(1)}) \quad (2.24)$$

and coincides with (2.13), provided that the Yang-Mills constant is $g = \sqrt{2}m$.
The ansatz for the 11 dimensional 3-index tensor $A_{\Pi\Sigma\Omega}$ given in [6] differs from the one of Lü and Pope. However, since it is defined only up to a gauge transformation (namely up to $\partial_{[\Pi}\Lambda_{\Sigma\Omega]}$) we will compare the ansatz for its field strength $F_{(4),11} = dA_{(4),11}$, i.e. (2.4) vs. (2.14). The two ansätze will turn out to be the same if we assume that the three form $A_{(3)}$ is a linear combination of $S_{(3)}$ and $\omega_{(3)}$.
To show the equivalence of the two ansätze, we need to make use of the following identities involving the spherical harmonics:

$$\begin{aligned}
\epsilon_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}} D\hat{Y}^{\hat{\mu}} \wedge D\hat{Y}^{\hat{\nu}} \wedge D\hat{Y}^{\hat{\rho}} \wedge D\hat{Y}^{\hat{\sigma}} &= 0 && (2.25)\\
\epsilon_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}} D\hat{Y}^{\hat{\mu}} \wedge D\hat{Y}^{\hat{\nu}} \wedge D\hat{Y}^{\hat{\rho}} \hat{Y}^{\hat{\sigma}} &= -\frac{1}{8}\epsilon_{ijk} h^i \wedge h^j \wedge h^k \equiv -\frac{3!}{8}\epsilon_{(3)} && (2.26)\\
\epsilon_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}} D\hat{Y}^{\hat{\mu}} \wedge D\hat{Y}^{\hat{\nu}} \wedge F^{\hat{\rho}\hat{\sigma}}_{(2)} &= -\frac{1}{2\sqrt{2}}\epsilon_{ijk} F^i_{(2)} \wedge h^j \wedge h^k && (2.27)\\
\epsilon_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}} F^{\hat{\mu}\hat{\nu}}_{(2)} \wedge D\hat{Y}^{\hat{\rho}} Y^{\hat{\sigma}} &= \frac{1}{\sqrt{2}} F^i_{(2)} h^i && (2.28)
\end{aligned}$$

To prove these identities, one needs to use the form of σ^i and $d\sigma^i$ in (2.21,2.22) and the embedding of the gauge fields in (2.15) (remember that $F^{\hat{\mu}\hat{\nu}}_{(2)} = 2(dB^{\hat{\mu}\hat{\nu}}_{(1)} + 2mB^{\hat{\mu}\hat{\rho}}_{(1)} \wedge B^{\hat{\rho}\hat{\nu}}_{(1)}))$. Then, one finds the various terms in (2.4) in terms of the left-hand side of these identities:

$$\epsilon_{A_1\ldots A_5} DY^{A_1} \wedge \ldots \wedge DY^{A_4} \frac{T\cdot Y^{A_5}}{Y\cdot T\cdot Y} = -\frac{3!}{2}\cos^3\xi d\xi \wedge \epsilon_{(3)} \quad (2.29)$$

$$\begin{aligned}
&\epsilon_{A_1\ldots A_5} DY^{A_1} \wedge DY^{A_2} \wedge DY^{A_3} \left(\frac{T\cdot Y^{A_4}}{Y\cdot T\cdot Y}\right) Y^{A_5} = \frac{3!}{2}\cos^3\xi \epsilon_{(3)} \wedge d\xi \tilde{\Delta}^{-2}\cos^3\xi\\
&\Big[3X\tilde{\Delta} - 2\sin^2\xi\cos^2\xi(X^{-8} - 2X^{-3} + X^2) + X^2\cos^2\xi\\
&+X^{-3}(\sin^4\xi\cos^3\xi + \cos^7\xi) + X^{-8}\sin^2\xi\cos^5\xi\Big] \quad (2.30)
\end{aligned}$$

$$\begin{aligned}
&\epsilon_{A_1\ldots A_5} F^{A_1A_2}_{(2)} \wedge DY^{A_3} \wedge DY^{A_4} \frac{T\cdot Y^{A_5}}{Y\cdot T\cdot Y}\\
&= 2\epsilon_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}} \left(\frac{\cos^2\xi\sin\xi X^{-4}}{\tilde{\Delta}} F^{\hat{\mu}\hat{\nu}}_{(2)} \wedge D\hat{Y}^{\hat{\rho}} \wedge D\hat{Y}^{\hat{\sigma}} + 2\cos\xi d\xi \wedge F^{\hat{\mu}\hat{\nu}}_{(2)} \wedge D\hat{Y}^{\hat{\rho}}\hat{Y}^{\hat{\sigma}}\right)
\end{aligned} \quad (2.31)$$

and one recovers most of (2.14). ‡

‡To get the same overall coeficient as in (2.14) we have to take into account that the three form potential of 11-d sugra used in [15] is related to the one used in [34, 5] by a rescaling with $6\sqrt{2}$, and that in the conventions of [15] $F_{\Lambda\Pi\Sigma\Omega}$ equals $4\partial_{[\Lambda}A_{\Pi\Sigma\Omega]}$, while for the authors of [34, 5] $F_{\Lambda\Pi\Sigma\Omega} = 24\partial_{[\Lambda}A_{\Pi\Sigma\Omega]}$. Also note that in [5] the form $F_{(4)}$ was not normalized: $F_{(4)} = F_{\Lambda\Pi\Sigma\Omega}dx^\Lambda \wedge dx^\Pi \wedge dx^\Sigma \wedge dx^\Omega$.

In the $\mathcal{N}=2$ model of [33] the lagrangean is quadratic in $dA_{(3)}$, and thus to reduce the on-shell number of degrees of freedom of $A_{(3)}$ one needs to supplement the field equations with the self-duality in odd dimensions constraint [32, 15]

$$X^4 \star F_{(4),7} = \frac{-1}{\sqrt{2}} g A_{(3),7} + \frac{1}{2}\omega_{(3)} \tag{2.32}$$

In fact, we should stress that the $\mathcal{N}=4$ model yields a first order field equation for $S_{(3)}$, which is the square root of the quadratic field equation of $A_{(3),7}$. We note that the field equations obtained from the truncated $\mathcal{N}=4$ action correspond to the field equations and constraints of the $\mathcal{N}=2$ model. This implies that one should use the following lagrangean for describing the bosonic sector of $\mathcal{N}=2$ $d=7$ gauged sugra with topological mass term [§]:

$$\begin{aligned}
\sqrt{g_7^{-1}} L_{7d,\mathcal{N}=2} \quad = \quad & R + g^2(2X^{-3} + 2X^2 - X^{-8}) \\
& +5\partial_\alpha X^{-1}\partial_\alpha X + \frac{g^2}{2} X^{-4} S_{\alpha\beta\gamma} S^{\alpha\beta\gamma} \\
& -\frac{1}{4} X^{-2} F^i_{\alpha\beta} F^{\alpha\beta\, i} + \epsilon^{\alpha\beta\gamma\delta\epsilon\eta\zeta}\sqrt{g_7^{-1}} \left(\frac{g}{24\sqrt{2}} S_{\alpha\beta\gamma} F_{\delta\epsilon\eta\zeta} \right. \\
& \left. + (-\frac{1}{48\sqrt{2}g}\omega_{\alpha\beta\gamma} + \frac{i}{8\sqrt{3}} S_{\alpha\beta\gamma}) F^i_{\delta\epsilon} F^i_{\eta\zeta} \right)
\end{aligned} \tag{2.33}$$

The field equation for $S_{\alpha\beta\gamma}$ reads

$$-g^2 X^4 S_{(3)} = \star d(\sqrt{2} g S_{(3)} + \frac{i}{2\sqrt{3}}\omega_3) \tag{2.34}$$

and coincides with the self-duality constraint (2.32) if

$$S_{(3)} = \frac{i}{2\sqrt{3}} \left(A_{(3),7} - \frac{\omega_{(3)}}{\sqrt{2}g} \right) \tag{2.35}$$

On the other hand, in $F_{(4)}$ (2.4), the remaining terms which at the first sight seem to differ from the ones in (2.14) are

$$\begin{aligned}
& \frac{3}{\sqrt{2}} \left(\frac{-8i}{\sqrt{3}} d(S_{(3)} \sin\xi) + \sqrt{2} g \sin\xi F^i_{(2)} \wedge F^i_{(2)} \right) \\
& = \frac{3}{\sqrt{2}} \left(\frac{-8i}{\sqrt{6}} \sin\xi d(\sqrt{2} S_{(3)} + \frac{i}{2\sqrt{3}}\omega_{(3)}) + \frac{8i}{\sqrt{3}} S_{(3)} \cos\xi \wedge d\xi \right) \qquad (2.36) \\
& = \frac{3}{\sqrt{2}} \left(\frac{8}{6} dA_{(3),7} \sin\xi + \frac{8}{3g\sqrt{2}} \cos\xi X^4 \star F_{(4),7} \wedge d\xi \right) \qquad (2.37)
\end{aligned}$$

where to go from (2.36) to (2.37) we made use of the self-duality constraint (2.32) which is part of the truncation procedure for [15]. Thus we were able to completely recover the ansatz (2.14) from our ansatz (2.4). This concludes that indeed, there exists a consistent truncation of the maximal gauged 7d sugra to $\mathcal{N}=2$ gauged $(SU(2))$ 7d sugra with topological mass term.

[§]Note that the limit $g \to 0$ is singular, as it was in the $\mathcal{N}=4$ gauged sugra model.

Finally, by taking the singular limit $S_4 \to S_3 \times \mathbf{R}$ in the same way as in [14], we recover the ansatz of Chamseddine and Sabra [22]. They proposed an ansatz for obtaining the $\mathcal{N} = 2$, $d = 7$ gauged sugra, with no topological mass term ($m = 0$) for the three form potential $A_{(3),7}$, and checked that the action of lower dimensional theory is reproduced from the 11-dimensional sugra action, when compactifying on $S_3 \times S_1$. This, however, does not generally guarantee consistency of the KK truncation. Since we get the ansatz of Chamseddine and Sabra as a limiting case of an ansatz whose consistency was rigorously proven, we conclude that their ansatz is also consistent.

We begin with S_4 written as before as a foliation of S_3 with latitude coordinate ξ. Then, we redefine

$$\xi = a\xi' \tag{2.38}$$

and take the limit $a \to 0^+$ which will yield an unrestricted ξ', taking values on the real line. Then $\tilde{\Delta} \to X$, and taking the limit $a \to 0$ in (2.13), we get

$$ds_{11}^2 = X^{1/3} ds_7^2 + 2a^2 g^{-2} X^{10/3} d\xi'^2 + \frac{1}{2} g^{-2} X^{-5/3} \sum_i (\sigma^i - gA^i_{(1)})^2 \tag{2.39}$$

We also redefine the fields and the coupling constant

$$\begin{aligned} X &= X' a^{-2/5} \\ g &= g' a^{2/5} \\ A_{(1)} &= A'_{(1)} a^{-2/5} \end{aligned} \tag{2.40}$$

in order to produce only an overall dependence on a in ds_{11}^2

$$ds_{11}^2 = a^{-2/15} \left(X'^{1/3} ds_7^2 + 2g'^{-2} X'^{10/3} d\xi'^2 + \frac{1}{2} g'^{-2} X'^{-5/3} \sum_i (\sigma^i - gA^i_{(1)})^2 \right) \tag{2.41}$$

At this moment, noticing that the metric has a Killing vector $\frac{\partial}{\partial \xi'}$ we decide to reinterpret ξ' as an angular coordinate on S_1. With a similar analysis for $F_{(4),11}$ given in (2.14), and with the redefinition

$$F_{(4),7} = F'_{(4),7} a^{4/5} \tag{2.42}$$

we get

$$\begin{aligned} F_{(4),11} &= a^{-1/5} \left(\frac{g'^{-3}}{\sqrt{2}} d\xi' \wedge \epsilon_{(3)} + \sqrt{2} g'^{-1} X'^4 \star F'_{(4),7} \wedge d\xi' \right. \\ &\left. - \frac{1}{\sqrt{2}} g'^{-2} F'^i_{(2)} \wedge d\xi' \wedge h^i \right) \\ &= a^{-1/5} \frac{g'^{-1}}{\sqrt{2}} d\xi' \wedge \left(g'^{-2} \epsilon_{(3)} - 2X'^4 * F'_{(4),7} - g'^{-1} F'^i_{(2)} \wedge h^i \right) \end{aligned} \tag{2.43}$$

To get rid of the a dependence in the metric and $F_{(4),11}$ ansatze we exploit the scaling symmetry of the 11-dimensional sugra equations of motion [14, 35]

$$ds_{11}^2 \to ds_{11} k^2 \tag{2.44}$$

$$F_{(4),11} \to F_{(4),11} k^3 \tag{2.45}$$

The relation between the scalar field X' and the dilaton of [22] is $X' = \exp 4\hat{\phi}/5$. Substituting it in (2.41, 2.43) and we are led to the ansatz of Chamseddine and Sabra

$$\begin{aligned}
ds^2_{11} &= e^{4\hat{\phi}/15}ds^2_7 + \frac{1}{4}e^{-4\hat{\phi}/3}(\sigma^i - \sqrt{2}A^i_{(1)})^2 + e^{8\hat{\phi}/3}d\xi^2 \\
F_{(4),11} &= d\xi \wedge \Big[dB_{(2)} - (A^i_{(1)} \wedge dA^i_{(1)} + \frac{1}{3}\epsilon_{ijk}A^i_{(1)} \wedge A^j_{(1)} \wedge A^k_{(1)}) \\
&\quad -\frac{1}{\sqrt{2}}dA^i_{(1)} \wedge \sigma^i + \frac{1}{4}\epsilon_{ijk}A^i_{(1)} \wedge \sigma^j \wedge \sigma^k + \frac{1}{12\sqrt{2}}\epsilon_{ijk}\sigma^i \wedge \sigma^j \wedge \sigma^k\Big]
\end{aligned} \tag{2.46}$$

where the two form $B_{(2)}$ is related to the three form $A'_{(3),7}$ through a duality transformation, and to reach the conventions of Chamseddine and Sabra, we set $g' = \sqrt{2}$.

- graviton, two abelian gauge fields $A^i_{(1)}$ and two scalars X_i, i=1,2
  This model was discussed in [13] and it allows to interpret 2-charge AdS black holes as the decoupling limit of rotating M2-branes. The ansatz for a consistent bosonic truncation of $d = 11$ sugra to this subset of bosonic fields reads:

$$\begin{aligned}
ds_{11} &= \tilde{\Delta}^{1/3}ds^2_7 + g^-2\tilde{\Delta}^{-2/3}\left(X_0^{-1}d\mu_0^2 + \sum_{i=1}^{2} X_i^{-1}(d\mu_i^2 + \mu_i^2(d\phi + gA^i_{(1)})^2)\right) \\
\hat{\star}F_{(4),11} &= 2g\sum_{i=0}^{2}(X_i^2\mu_i^2 - \tilde{\Delta}X_i)\epsilon_{(7)} + g\tilde{\Delta}X_0\epsilon_{(7)} + \frac{1}{2g}\sum_{i=0}^{2} X_{{}_1}^{-1} \star dX_i \wedge d(\mu_i^2) \\
&\quad +\frac{1}{2g^2}\sum_{i=1}^{2} X_i^{-2}d(\mu_i^2) \wedge (d\phi + gA^i_{(1)}) \wedge \star F^i_{(2)}
\end{aligned} \tag{2.47}$$

where $X_0 = (X_1X_2)^{-2}$, $\mu_i, i = 0, i, 2$, and $\phi_i, i = 1, 2$ parametrize the 4-sphere, and $\tilde{\Delta} = \sum_{i=0}^{2} X_i\mu_i^2$. The relationship between the Cartesian coordinates Y^A, $A = 1, \ldots, 5$ and the new variables μ_i and ϕ_i is given by

$$\begin{aligned}
Y_5 &= \mu_0 \\
Y_1 &= \mu_1 \sin\phi_1 \qquad Y_2 = \mu_1\cos\phi_1 \\
Y_3 &= \mu_2 \sin\phi_2 \qquad Y_4 = \mu_2\cos\phi_2
\end{aligned} \tag{2.48}$$

where the constraint $Y^AY^A = 1$ translates into $\mu_0^2 + \mu_1^2 + \mu_2^2 = 1$. The embedding of the scalar fields in the symmetric matrix T_{AB} of the maximal $d = 7$ gauged sugra is

$$T_{AB} = diag(X_1, X_1, X_2, X_2, X_0) \tag{2.49}$$

Fianlly, the abelian gauge fields $A^i_{(1)}, i = 1, 2$ are in the Cartan subalgebra of $SO(5)_g$:

$$\begin{aligned}
\frac{1}{\sqrt{2}}A^1_{(1)} &= -\left(B^{12}_{(1)} + B^{34}_{(1)}\right) \\
\frac{1}{\sqrt{2}}A^2_{(1)} &= B^{12}_{(1)} - B^{34}_{(1)}
\end{aligned} \tag{2.50}$$

3 The metric ansatz

We will now analyze the case when we reduce a certain sugra in D dimensions on a sphere $S_n = SO(n+1)/SO(n)$ and see what we can learn. In order that the sphere is a background solution for the sugra of the spontaneous compactification type, we need an antisymmetric tensor $F_{(n)}$, such that this background is of the Freund-Rubin type, i.e.

$$F_{\mu_1...\mu_n} = \sqrt{\overset{\circ}{g}}\epsilon_{\mu_1...\mu_n} \tag{3.1}$$

We note that for the compactification of 11d sugra on $AdS_4 \times S_7$, one has a $F_{(4)}$, but we would take the dual of this, namely an $F_{(7)}$. So, we have to start with an action

$$S^{(D)} = \int d^D x\sqrt{g}(R^{(D)} + F^2_{(n)} + ...) \tag{3.2}$$

where ... contain other fields which we put to zero. A first observation is that this starting point is not the most general. If one would add a dilaton coupling

$$S^{(D)} = \int d^D x\sqrt{g}(R^{(D)} + e^{a(D,n)\phi}F^2_{(n)} + b(D)(\partial\phi)^2 + ...) \tag{3.3}$$

one would get the most general starting point for a supergravity action admitting $n-2$ brane solutions. Then the near-horizon of that p-brane will be an $AdS_{D-n} \times S_n$ background solution of the theory, but the dilaton would not be constant in that case. We leave the generalization to this (more interesting) case to future work. But the action in (3.2) has been used already in [25], so it is not without interest to consider it.

We first ask what is the compactification ansatz if in the lower dimension d=D-n we keep only scalars. We know that the d-dimensional sugra which is obtained by compactification is a *gauged* sugra, because the sphere S_n has isometry group $SO(n+1)$, which becomes the gauge group $SO(n+1)_g$ of the d dimensional sugra. From the known examples of gauged 4d sugra, obtained as 11d sugra on $AdS_4 \times S_7$, 7d gauged sugra as 11d sugra on $AdS_7 \times S_4$, and the less understood case of 5d gauged sugra as 10d IIB sugra on $AdS_5 \times S_5$, we know that the scalar fields are in a coset G/H, where G is a global isometry of the ungauged model which is broken down to $SO(n+1)_g$ by the gauging, and H is a local composite symmetry. The gravitinos of the gauged sugra are in a fundamental spinorial representation of this group H.

Moreover, we know that one way to determine which coset we have is to count the number of scalar degrees of freedom and the number of gravitinos and to try to match a coset G/H which does the job. The number of scalar degrees of freedom is always the same as for the torus compactification (going from the torus to the sphere is the same as going from the ungauged to the gauged model). So the minimum number of scalar degrees of freedom is given by the following. The metric will have n(n+1)/2 scalar d.o.f. from the compact space metric. The antisymmetric tensor will have at least n scalar d.o.f., coming from the component $A_{\mu_1...\mu_{n-1}}$, with μ_i compact space indices. It could have more, if one of the components with spacetime indices can be dualized to a scalar. That doesn't happen if $A_{\alpha_1...\alpha_{n-1}}$ is the d-dimensional dual of a form A_k with $k > 0$, i.e. if $d-n-1 > 0$. This is the case of the $AdS_7 \times S_4$ background, for instance. In such a case, the coset will be an n(n+1)/2 +n=n(n+3)/2 dimensional manifold.

This is exactly the dimension of $Sl(n+1)/SO(n+1)_c$ ($(n+1)^2-1-n(n+1)/2$). Also, we know that the gravitinos multiply Killing spinors in the linearized KK reduction ansatz (we have something of the type $\psi_{\alpha I}\eta^I$, as we will see later). That means that it is safe to assume in that case that the fermions are in a spinor representation of $SO(n+1)_c$ (because I is a spinor index for $SO(n+1)_g$).

So, we can say that the scalar coset in the case $d > n+1$ is determined, namely $Sl(n+1)/SO(n)$. In the other cases, the scalar manifold will be bigger, but $Sl(n+1)/SO(n)$ will still be a submanifold, so we will restrict to it in the general case. By a straightforward extension of the $AdS_7 \times S_4$ case, where the coset is $Sl(5)/SO(5)$, a coset element will be a matrix $\Pi_A{}^i$, where A is an $SO(n+1)_g$ index and i is an $SO(n+1)_c$ index. The n(n+3)/2 physical scalars can be described by the matrix $T^{AB} = \Pi_i^{-1A}\Pi_i^{-1B}$, which is symmetric and has unit determinant. The d-dimensional gauged sugra will have gravity+scalar terms:

$$S^{(d)} = \int d^d x \sqrt{\overset{\circ}{g}}(R^{(d)} + P_{\alpha ij}^2 - V(T)) \tag{3.4}$$

where $P_{\alpha ij}$ is the symmetric part of $(\Pi^{-1})_i^A \delta_A{}^B \partial_\alpha \Pi_B{}^k \delta_{kj}$ (where the gauge field was put to zero), and so the kinetic term in (3.4) is $1/8Tr(\partial_\alpha T^{-1}\partial^\alpha T)$. The scalar potential is $V(T) = g^2/4(T^2 - 2T_{AB}^2)$. We will find an ansatz for the embedding of (3.4) into (3.2), and see how to extend it for the case that in (3.4) we keep also the gauge fields of $SO(n+1)_g$. We note here that the problem of dimensional reduction to gravity + scalars was analyzed also in [16]. But the authors of [16] looked only at the cases $AdS_4 \times S_7$, $AdS_7 \times S_4$ and $AdS_5 \times S_5$, moreover they restricted themselves to a diagonal T matrix. That is OK as long as gauge fields don't couple to scalars, but we want to explore how to generalize to the case where gauge fields are also nonzero.

Guided by the ansatz in [5, 6], we expect that the metric $g_{\mu\nu}$ on the compact space has the form

$$g_{\mu\nu} = \Delta^\beta \partial_\mu Y^A \partial_\nu Y^B T_{AB}^{-1} \tag{3.5}$$

where Δ is $\sqrt{\det g_{\mu\nu}/\det \overset{\circ}{g}_{\mu\nu}}$, with the inverse metric given by

$$g^{\mu\nu} = \frac{\Delta^{-\beta}}{Y \cdot T \cdot Y}\partial^\mu Y_A \partial^\nu Y_B \left(T^{AB} - \frac{(T\cdot Y)^A (T\cdot Y)^B}{Y\cdot T\cdot Y}\right) \tag{3.6}$$

But

$$\begin{aligned}
&\det \tilde{g}_{\mu\nu} \equiv \det T_{AB}^{-1}\partial^\mu Y^A \partial^\nu Y^B \equiv \epsilon^{\mu_1\dots\mu_n}\epsilon^{\mu_1'\dots\mu_n'}\tilde{g}_{\mu_1\mu_1'}\dots\tilde{g}_{\mu_n\mu_n'}\\
&= \epsilon^{\mu_1\dots\mu_n}\partial_{\mu_1}Y^{A_1}\dots\partial_{\mu_n}Y^{A_n}\epsilon^{\nu_1\dots\nu_n}\partial_{\nu_1}Y^{B_1}\dots\partial_{\nu_n}Y^{B_n}T^{-1}_{A_1B_1}\dots T^{-1}_{A_nB_n}\\
&= \overset{\circ}{g}\, \epsilon^{A_1\dots A_{n+1}}\epsilon^{B_1\dots B_{n+1}}Y_{A_{n+1}}Y_{B_{n+1}}T^{-1}_{A_1B_1}\dots T^{-1}_{A_nB_n} = \overset{\circ}{g}\, T^{A_{n+1}B_{n+1}}Y_{A_{n+1}}Y_{B_{n+1}}
\end{aligned} \tag{3.7}$$

where to get from the second to the third line we have used a simple generalization of the relations in [6] and the fact that $detT = 1$. Now we can see that

$$\Delta^{2-\beta n} = T^{AB}Y_A Y_B \tag{3.8}$$

We take a standard ansatz for the vielbein, keeping the gauge fields, of the type

$$E^a_\alpha = \Delta^{-\frac{1}{d-2}} e^a_\alpha(y) \quad , \quad E^m_\alpha = B^\mu_\alpha E^m_\mu \tag{3.9}$$

$$E^a_\mu = 0 \quad , \quad E^m_\mu = \frac{1}{g} e^m_\mu(x,y) \tag{3.10}$$

with $B^\mu_\alpha = B^{AB}_\alpha V^\mu_{AB}$, where B^{AB}_α is the $SO(n+1)_g$ gauge field and V^{AB}_μ is the Killing vector. Then the D-dimensional line element becomes

$$\begin{aligned} ds^2_D &= \Delta^\alpha ds^2_d + g^{-2}\Delta^\beta T^{-1}_{AB} DY^A DY^B \\ &= \Delta^{-\frac{2}{d-2}} ds^2_d + g^{-2}\Delta^\beta T^{-1}_{AB} DY^A DY^B \end{aligned} \tag{3.11}$$

where $DY^A \equiv dY^A + gB^{AB}Y_B$ is a covariant derivative. We note that the factor $\Delta^{-\frac{2}{d-2}}$ is necessary to obtain the correct Einstein action in d dimensions. Next we will determine β from the requirement that we recover the correct scalar potential $V(T)$ after integrating the Einstein action $\int d^D x R^{(D)}$ and the kinetic term of the n-form field $\int dx^D F^2_{(n)}$.

First, we compute the contribution coming from the integration of the Einstein action. In what follows we set (for simplicity reasons) the Yang-Mills coupling constant to $g = 1$. Using the metric ansatz given in (3.11) we get

$$\begin{aligned} \int \sqrt{g^{(d)}}\sqrt{\overset{\circ}{g}{}^{(n)}}\Delta^{\alpha d/2+1} d^D x R^{(D)} = \int \sqrt{g^{(d)}}\sqrt{\overset{\circ}{g}{}^{(n)}} d^D x \Delta^\alpha \Big(R^{(n)} \\ + \frac{d}{2}\left(\frac{d-3}{2} + \frac{2}{\alpha}\right)\Delta^{-2\alpha}\partial_\mu \Delta^\alpha \partial_\nu \Delta^\alpha g^{\mu\nu}\Big) \end{aligned} \tag{3.12}$$

where we already used that $\alpha d/2 + 1 = \alpha$. Moreover, if we write

$$g_{\mu\nu} = \Delta^\beta \tilde{g}_{\mu\nu} \tag{3.13}$$

we can use the following formula for the relation between the Ricci scalars of two metrics related by a conformal rescaling as in (3.13)

$$R^{(n)} = \tilde{R}^{(n)}\Delta^{-\beta} - \frac{1}{4}(n-1)(n-2)g^{\mu\nu}(\partial_\mu \ln \Delta^{-\beta})(\partial_\nu ln\Delta^{-\beta}) - (n-1)g^{\mu\nu}D_\mu\partial_\nu \ln \Delta^{-\beta} \tag{3.14}$$

For our particular metric $\tilde{g}_{\mu\nu} = \partial_\mu Y \cdot T \cdot \partial_\nu Y$, the Ricci scalar has the following expression (independent of the dimensionality of the compact space):

$$\tilde{R}^{(n)} = \frac{-2Y_AY_B(T^3)^{AB} + 2TY_AY_B(T^2)^{AB} + Y_AY_BT^{AB}(Tr(T^2) - T^2)}{(Y_AY_BT^{AB})^2} \tag{3.15}$$

And then we obtain finally

$$\begin{aligned} \int \sqrt{g^{(d)}}\sqrt{\overset{\circ}{g}{}^{(n)}}\Delta^\alpha d^D x R^{(D)} = \int \sqrt{g^{(d)}}\sqrt{\overset{\circ}{g}{}^{(n)}} d^D x \Delta^{\alpha-\beta}\Big\{\tilde{R}^{(n)} - \frac{4}{(2-\beta n)^2}\Big[\frac{d}{2}\Big(\frac{d-3}{2} \\ + \frac{2}{\alpha}\Big)\alpha^2 - \frac{(n-1)(n-2)\beta^2}{4} - \beta(\alpha-1)(n-1)\Big]\left[\frac{(Y\cdot T^2\cdot Y)^2}{(Y\cdot T\cdot Y)^3} - \frac{Y\cdot T^3\cdot Y}{(Y\cdot T\cdot Y)^2}\right]\Big\} \end{aligned} \tag{3.16}$$

and we can already see that we need to have

$$\Delta^{\alpha-\beta} = Y \cdot T \cdot Y \tag{3.17}$$

in order to obtain terms of the type T^2 in d dimensions. That gives a solution for β as

$$\beta = \frac{2}{n-1}\frac{d-1}{d-2} \tag{3.18}$$

And now we can write the full line element as

$$ds_D^2 = (Y \cdot T \cdot Y)^{-\frac{2}{(d-2)(2-\beta n)}} ds_7^2 + (Y \cdot T \cdot Y)^{\frac{\beta}{2-\beta n}} T^{-1}_{AB} DY^A DY^B \tag{3.19}$$

with β found before. We can compare with the ansatz given in [16] for the case of no gauge fields and a diagonal matrix $T_{AB} = X_A \delta_{AB}$, valid for the $AdS_4 \times S_7, AdS_7 \times S_4$ and $AdS_5 \times S_5$compactifications, namely

$$ds_D^2 = (X_A Y_A^2)^{\frac{2}{d-1}} ds_d^2 + (X_A Y_A^2)^{-\frac{d-3}{d-1}} (X_A^{-1} dY_A^2) \tag{3.20}$$

By comparing the two results, we see that we have the same result if $d(n-3) = 3n-5$, an equation which has as the ONLY solutions ($d=4, n=7$), ($d=7, n=4$) and ($d=n=5$). So we have the curious fact that the formula derived in [16], although is written in a general form (with a general d), is only valid in the cases for which it was derived, not for general d. For general d and n, one should take the appropriate truncation of our formula (3.19).

Finally, we will complete the calculation of the scalar potential. We can compute the integrals on the sphere using the formulas

$$\int \frac{Y^A Y^B}{Y \cdot T \cdot Y} = \frac{1}{n+1} T^{-1}_{AB} \tag{3.21}$$

$$\begin{aligned} \int \frac{Y^A Y^B Y^C Y^D}{(Y \cdot T \cdot Y)^2} &= \frac{1}{(n+1)(n+3)} (T^{-1}_{AB} T^{-1}_{CD} \\ &\quad + T^{-1}_{AC} T^{-1}_{BD} + T^{-1}_{AD} T^{-1}_{BC}), \quad \text{etc.} \end{aligned} \tag{3.22}$$

From that we derive, for instance

$$\int \left[\frac{Y \cdot T^3 \cdot Y}{Y \cdot T \cdot Y} - \left(\frac{Y \cdot T^2 \cdot Y}{Y \cdot T \cdot Y} \right)^2 \right] = \frac{1}{n+3} \left(T^2_{AB} - \frac{T^2}{n+1} \right) \tag{3.23}$$

In conclusion, the Einstein term in the D-dimensional action gives the following contribution to d-dimensional the scalar potential

$$(n-1) \int \sqrt{g^{(d)}} d^d x \left[\frac{d-1}{d+n-2} \frac{1}{n+3} (T^2_{AB} - \frac{T^2}{n+1}) + \frac{1}{n+1} (T^2_{AB} - T^2) \right] \tag{3.24}$$

We will now turn to the ansatz for the antisymmetric tensor field. We will again generalize the result in [6], where the ansatz for the 3-index tensor in the presence of only scalar fields is

$$A_{(3)} = -\frac{1}{6\sqrt{2}} \epsilon_{A_1 \ldots A_5} dY^{A_1} \wedge dY^{A_2} \wedge dY^{A_3} Y^{A_4} \left(\frac{T \cdot Y}{Y \cdot T \cdot Y} \right)^{A_5} \tag{3.25}$$

We write, similarly, an ansatz for the $n-1$ form

$$A_{(n-1)} = -a\epsilon_{A_1...A_{n+1}} dY^{A_1} \wedge ... \wedge dY^{A_{n-1}} Y^{A_n} \left(\frac{T\cdot Y}{Y\cdot T\cdot Y}\right)^{A_{n+1}} + \\ -\frac{1}{n}\frac{\overset{\circ}{D}_{\mu_n}}{\overset{\circ}{\Box}}(\epsilon_{\mu_1...\mu_n}\sqrt{\overset{\circ}{g}}) \tag{3.26}$$

where we kept only one factor of $T\cdot Y/(Y\cdot T\cdot Y)$ because we want to have an d-dimensional action involving only T^2 terms (and no TrT^{-1} terms, for example), completely analog to the case considered in [6]. Here a is a constant to be fixed. However, if we look then at the terms with only scalars and no derivative in $F_{(n)}$, we have

$$F_{(n)}|_{no\ \partial T} = (\sqrt{\overset{\circ}{g}}\epsilon_{\mu_1...\mu_n} dx^{\mu_1}...dx^{\mu_n})\,(1 \\ +\frac{a}{n}\left[\frac{T}{Y\cdot T\cdot Y} - (n+1) - 2\left(\frac{Y\cdot T^2\cdot Y}{(Y\cdot T\cdot Y)^2} - 1\right)\right]) \tag{3.27}$$

and then the term in the action becomes

$$F^2_{(n)}\Delta^\alpha|_{no\ \partial T} = \Delta^{\alpha-2} n!\left[1 - \frac{a(n-1)}{n} + \frac{a}{n}\left(\frac{T}{Y\cdot T\cdot Y} - 2\frac{Y\cdot T^2\cdot Y}{(Y\cdot T\cdot Y)^2}\right)\right]^2 \tag{3.28}$$

But in order to get only T^2 terms in the action, we need that the the power of Δ is $2(2-\beta n)$, so that we obtain a factor $(Y\cdot T\cdot Y)^2$ multiplying the bracket in (3.28). That gives again the equation $d(n-3) = 3n-5$, which is solved only by $(d=4, n=7)$, $(d=7, n=4)$ and $(d=n=5)$.

But we can easily see how we can modify the ansatz. Just multiply $F_{(n)\mu_1...\mu_n}$ by $\Delta^{3-\beta n-\frac{\alpha}{2}}$. Then $A_{(n-1)}$ will be

$$A_{(n-1)} = -\Delta^{3-\beta n-\frac{\alpha}{2}} a\epsilon_{A_1...A_{n+1}} dY^{A_1} \wedge ... \wedge dY^{A_{n-1}} Y^{A_n} \left(\frac{T\cdot Y}{Y\cdot T\cdot Y}\right)^{A_{n+1}} + \\ -\frac{1}{n}\frac{\overset{\circ}{D}_{\mu_n}}{\overset{\circ}{\Box}}(\epsilon_{\mu_1...\mu_n}\sqrt{\overset{\circ}{g}}\Delta^{3-\beta n-\frac{\alpha}{2}}) \tag{3.29}$$

Notice that we could have put $\Delta^{2-\beta n-\frac{\alpha}{2}}$ outside $\frac{\overset{\circ}{D}_{\mu_n}}{\overset{\circ}{\Box}}$, but that would generate an extra term in the scalar potential. Now the term in the action becomes

$$F^2_{(n)}\Delta^\alpha|_{no\ \partial T} = n!\int (Y\cdot T\cdot Y)\left[1 - \frac{a(n-1)}{n} + \frac{a}{n}\left(\frac{T}{Y\cdot T\cdot Y} - 2\frac{Y\cdot T^2\cdot Y}{(Y\cdot T\cdot Y)^2}\right)\right. \\ \left. +2a\left(1 - \frac{\alpha-2}{2(2-\beta n)}\right)\left(1 - \frac{Y\cdot T^2\cdot Y}{(Y\cdot T\cdot Y)^2}\right)\right]^2 \tag{3.30}$$

and by integration we get a combination of T^2_{AB} and T^2 terms, with a free parameter (a). Then a is fixed by requiring that in the sum of (3.30) and (3.24) the relative coefficient of T^2_{AB} and T^2 is -2 (the overall coeffiecient depends on the coupling constant g). It is not clear how one gets the correct kinetic terms fot T, since $\partial_\alpha A_{(n-1)\beta\gamma\delta}$ will contain

$\frac{\overset{\circ}{D}_{\mu n}}{\overset{\circ}{\Box}}$ terms. When integrated on the sphere, they should give contributions to the $P^2_{\alpha ij}$ kinetic terms too.

In the remainder of this section we will look at applications of our metric ansatz (3.19). Let us see that it can be applied for the $AdS_7 \times S_4$ case. The metric deduced by de Wit and Nicolai [2] is

$$\begin{aligned} \Delta^{-1} g^{\mu\nu} &= \frac{1}{8} V^{\mu IJ} V^{\nu KL} w_{i'j'}{}^{IJ} w^{i'j'}{}_{KL} \\ w_{ij}{}^{IJ} &= = u_{ij}{}^{IJ} + v_{ijIJ} \;\; ; \;\; w^{ij}{}_{KL} = u^{ij}{}_{KL} + v^{ijKL} \end{aligned} \tag{3.31}$$

where I,J,... are spinorial $SO(8)_g$ indices, and i',j',... are $SU(8)_c$ indices, both [i'j'] and [IJ] are antisymmetrized, and u and v together form a representation of E_7. Then one can define

$$V^{\mu AB} = (\Gamma^{AB})^{IJ} V^\mu_{IJ} \tag{3.32}$$

$$w_{ij}{}^{AB} = \frac{1}{64} (\Gamma^{AB})_{IJ} w_{ij}{}^{IJ} \tag{3.33}$$

where A,B,... are vector $SO(8)_g$ indices, and $V^{AB}_\mu = Y^A \partial_\mu Y^B$ and we used that $(\Gamma^{AB})_{IJ}(\Gamma_{AB})^{KL} = 16 \delta^{KL}_{IJ}$. Then if one restricts the scalar coset from $E_7/SU(8)_c$ to $Sl(8)/SO(8)_c$, with $SO(8)_g$ embedded in $Sl(8)$, the matrix w will reduce to

$$w_{ij}{}^{AB} = \Pi^{-1}{}_{[i}^{A} \Pi^{-1}{}_{j]}^{B} \tag{3.34}$$

and

$$w_{i'j'}{}^{AB} w^{i'j'}{}_{CD} = T^{[A}_{[C} T^{B]}_{D]} \tag{3.35}$$

and then we recover our general formula (3.6).

As a small application, we note that now we can write full line element as

$$ds^2_{11} = \Delta^{-1} ds^2_7 + \Delta^{-1} M_{AB} DY^A DY^B \tag{3.36}$$

where M_{AB} is the 'inverse' of $\tilde{M}^{AB} = w_{i'j'}{}^{AC} w^{i'j'}{}_{BD} Y_C Y^D$, i.e.

$$\tilde{M}^{AC} M_{BC} = \delta^A_B + (Y_B \text{terms}) \tag{3.37}$$

Note that $\tilde{M}$ has no inverse in the usual sense, because it has an eigenvector with zero eigenvalue, namely Y_A, but we only need (3.37) to get (3.36).

A new application for our method is an ansatz for the metric in the $AdS_5 \times S_5$ case. We guess it by the inverse procedure to the one described for the $AdS_4 \times S_7$ case. Namely, we replace $\Pi^{-1}{}_{[i}^{A} \Pi^{-1}{}_{j]}^{B}$ by the projection of the $E_6/USp(8)_c$ scalar coset vielbein onto the **15** representation of $SO(6)_g$,

$$\Pi^{-1}{}_{[i}^{A} \Pi^{-1}{}_{j]}^{B} \to (\Pi^{-1}(\mathbf{15}))_{ab}{}^{AB} \equiv (\Gamma^{AB})_{\alpha\beta} P(\mathbf{15})^{\alpha\beta}{}_{\gamma\delta} (\Pi^{-1})_{ab}{}^{\gamma\delta} \tag{3.38}$$

In this way we arrive at the ansatz

$$\Delta^{-2/3} g^{\mu\nu} = V^\mu_{AB} V^\nu_{CD} (\Pi^{-1}(\mathbf{15}))_{ab}{}^{AB} (\Pi^{-1}(\mathbf{15}))_{ab}{}^{CD} \tag{3.39}$$

as a natural extension of our particular ansatz

$$\Delta^{-2/3}g^{\mu\nu} = V^{\mu}_{AB}V^{\nu}_{CD}T_{AC}T_{BD} \tag{3.40}$$

We will give another argument for (3.39) in the following section, by first computing the vielbein.

We note that we can again put the 10d line element in a form similar to (3.36), namely:

$$ds^2_{10} = \Delta^{-\frac{2}{d-2}}ds^2_5 + \Delta^{-2/3}N_{AB}DY^ADY^B \tag{3.41}$$

where N_{AB} is the 'inverse' of $\tilde{N}_{AB} = (\Pi^{-1}(\mathbf{15}))_{ab}{}^{AC}Y_C(\Pi^{-1}(\mathbf{15}))_{ab}{}^{BD}Y_D$, i.e.

$$\tilde{N}^{AC}N_{BC} = \delta^A_B + (Y_B\text{terms}) \tag{3.42}$$

4 The fermion fields and vielbein ansatz

In this section we will try to give a general discussion of the fermionic fields and the vielbein in the case of compactifications on spheres.

For the vielbein, we already gave a general ansatz, namely

$$E^a_\alpha = \Delta^{-\frac{1}{d-2}}e^a_\alpha \quad , \quad E^m_\alpha = B^\mu_\alpha E^m_\mu \tag{4.1}$$

$$E^a_\mu = 0 \quad , \quad E^m_\mu \tag{4.2}$$

with $B^\mu_\alpha = B^{AB}_\alpha V^\mu_{AB}$, the only thing that remains to give is an ansatz for the compact space vielbein, E^m_μ. For that, we need first an ansatz for the fermions.

In a supergravity theory, we always have a gravitino field. By dimensional reduction, we obtain a set of gravitinos in d dimensions, transforming in the fundamental representation under the composite symmetry of the gauged sugra. For example, in the $AdS_7\times S_4$ case, we have gravitinos in the $\mathbf{4}$ of $SO(5)_c = USp(4)_c$. For the $AdS_4\times S_7$ case we have gravitinos in the $\mathbf{8}$ of $SU(8)_c$, and in the $AdS_5\times S_5$ case, the gravitinos are in the $\mathbf{8}$ of $USp(8)_c$. Throughout most of the discussion we will restrict ourselves to a $SO(n+1)_c$ composite symmetry group ($SO(5)_c, SO(8)_c\in SU(8)_c, SO(6)_c \in USp(8)_c$).

However, the usual (linearized) ansatz for the gravitinos involves a Killing spinor ($\Psi_\alpha(y,x) = \psi_{\alpha I}(y)\eta^I(x)$), which has an $SO(n+1)_g$ spinor index of the gauge field. That tells us that there should exits a matrix $U^{I'}{}_I$, with I' a composite group index, interpolating between the gravitinos and the Killing spinor.

The result we obtained in 7d in [5, 6] is

$$\begin{aligned}
\Psi_a &= \Delta^{1/10}(\gamma_5)^{1/2}\psi_a - \frac{1}{5}\tau_a\gamma_5\gamma^m\Delta^{1/10}(\gamma_5)^{-1/2}\psi_m && (4.3)\\
\Psi_m &= \Delta^{1/10}(\gamma_5)^{-1/2}\psi_m && (4.4)\\
\varepsilon &= \Delta^{-1/10}(\gamma_5)^{1/2}\epsilon, \bar{\varepsilon} = \Delta^{-1/10}\bar{\epsilon}(\gamma_5)^{1/2} &&\\
\psi_\alpha(y,x) &= \psi_{\alpha I'}(y)U^{I'}{}_I(y,x)\eta^I(x) \;\; where \;\; \psi_a = e^\alpha_a(y)\psi_\alpha(y,x) && (4.5)\\
\psi_m(y,x) &= \lambda_{J'K'L'}(y)U^{J'}{}_J(y,x)U^{K'}{}_K(y,x)U^{L'}{}_L(y,x)\eta^{JKL}_m(x) && (4.6)\\
\epsilon(y,x) &= \epsilon_{I'}(y)U^{I'}{}_I\eta^I(x) && (4.7)
\end{aligned}$$

We see that we needed a certain rotation to diagonalize the 7dimensional gravitino and spin 1/2 kinetic operators. In general, we expect that the rotation is more complicated, such that both Ψ_a and Ψ_m depend on both gravitino and spin 1/2 fields in d dimensions. Moreover, in general, the D dimensional sugra itself has spin 1/2 fields, so the rotation involved is probably more general than the one considered here. We will not have more to say about that, we will just note that the advertised matrix $U^{I'}{}_I$ appears in the ansatz for all the fermions.

One possible interpretation is that $U^{I'}{}_I$ gives a (field- and spherical harmonic-dependent) composite rotation. Indeed, U is a matrix in the coset $H_c/SO(n+1)_g$ (for instance because it has one index transforming under H_c and one transforming under $SO(n+1)_g$), so it can be interpreted as a field dependent H_c rotation. We note that if we break H_c to $SO(n+1)_c$ (we will do it for most of this section), then U becomes simply an $SO(n+1)$ matrix in the spinor representation. Since the vielbein couples to the fermions, we should expect that E^m_μ depends on U also, and we shall find exactly that. We note here that restricting U to be and $SO(n+1)$ matrix in the spinor representation means also that the fermions are restricted to lie in a spinor representation of $SO(n+1)$. For a concrete example, for the $AdS_4\times S_7$ case that would mean that the fermions, which are in a representation of SU(8) are restricted to transform only under an SO(8) representation.

We can however still say the following about the fermions. We take $\Gamma_a=\tau_a\times\gamma$, as the dimensional reduction of the gamma matrices from D to d dimensions (Γ_M to τ_a), where γ is one for odd-dimensional spheres and γ_{2n+1} for even-dimensional spheres. If both d and n are odd, the gamma matrix split involves an extra σ matrix, e.g. in $AdS_5\times S_5$ we have $\Gamma_a=\tau_a\times\mathbb{1}\times\sigma_1$ (and $\Gamma_m=\mathbb{1}\times\gamma_m\times(-\sigma_2)$). We will examine the $AdS_5\times S_5$ case separately. From the requirement that the D dimensional gravitino action reduces to the d dimensional one, more precisely that

$$\Delta^{-\frac{2}{d-2}}\Psi_a\Gamma^{ab\alpha}\partial_\alpha\Psi_b=\psi_{aI'}\tau^{ab\alpha}\partial_\alpha\psi_b^{I'}+... \tag{4.8}$$

(on the left-hand-side indices are flattened with the D-dimensional vielbein and on the right-hand-side with the d-dimensional one), we get the ansatz

$$\Psi_a(y,x)=\Delta^{\frac{1}{2(d-2)}}(\gamma)^p\psi_{aI'}U^{I'}{}_I(y,x)\eta^I(x)+\text{spin }\ 1/2\ \text{ terms} \tag{4.9}$$

where again Ψ_a is flattened with the D dimensional vielbein and ψ_a with the d-dimensional one. Moreover, we can derive the ansatz for the susy parameter,

$$\varepsilon(y,x)=\Delta^{-\frac{1}{2(d-2)}}(\gamma)^p\epsilon_{I'}U^{I'}{}_I\eta^I \tag{4.10}$$

Using this fermionic ansätze, we will derive the vielbein ansatz and then the relation that the matrix U has to satisfy.

The vielbein ansatz is easily derived from the susy law of the gauge fields. We have in general a susy law of the type

$$\delta B^{AB}_\alpha=(\Pi^{-1})_{I'J'}{}^{AB}(\bar\epsilon^{I'}\psi^{J'}_\alpha)+\text{spin }1/2\ \text{ terms} \tag{4.11}$$

That is so because a boson transforms into ϵ times a fermion, and by matching indices, it is easy to see that we need a bosonic matrix in a coset involving the gauge group (via

the A,B indices) and the composite group (via the I', J' indices). The only one available is a matrix $(\Pi^{-1})_{I'J'}{}^{AB}$ which is either the scalar coset vielbein, or some product of such vielbeins. We will treat separately the cases of $AdS_4 \times S_7, AdS_7 \times S_4$ and $AdS_5 \times S_5$, but in the case that we restrict the scalar manifold to the coset $Sl(n+1)/SO(n+1)$ (in particular, for $AdS_7 \times S_4$ there is no restriction made), we know that the susy law takes the form

$$\delta B_\alpha^{AB} = c\Pi_i^{-1A}\Pi_j^{-1B}(\gamma^{ij})_{I'J'}(\bar{\epsilon}^{I'}\psi_\alpha^{J'}) + \text{spin } 1/2 \text{ terms} \tag{4.12}$$

where c is a normalization constant. On the other hand, we can obtain the variation of B_α^μ starting from D dimensions, in a manner entirely analog to the one used in [6]. Namely, by assuming only that in D dimensions we have the following susy law for the vielbein

$$\delta E_\Lambda^M = \frac{1}{2}\bar{\varepsilon}\Gamma^M\Psi_\Lambda \tag{4.13}$$

and using the fact that in the gauge $E_\mu^a = 0$ we have a compensating off-diagonal Lorentz rotation $\Omega_m{}^a = -\Omega^a{}_m = -\frac{1}{2}\bar{\epsilon}\Gamma^a\Psi_m$, we get the variation of B_α^μ,

$$\begin{aligned}\delta B_\alpha^\mu &= \delta_{susy}(E_\alpha^m E_m^\mu) + E_\alpha^m(E_a^\mu\Omega^a{}_m) + (E_\alpha^a\Omega_a{}^m E_m^\mu \\ &= \frac{1}{2}E_\mu^m E_\alpha^a\bar{\varepsilon}[\Gamma_a\Psi^m + \Gamma^m\Psi_a]\end{aligned} \tag{4.14}$$

Now we can substitute the gravitino ansatz and (in the hypothesis that there is no d-dimensional gravitino contribution in Ψ_m), we get

$$\frac{s}{2}\Delta^{-\frac{1}{d-2}}E_m^\mu[\epsilon\gamma^m\psi_\alpha + \text{spin } 1/2 \text{ terms}] \tag{4.15}$$

where s is a phase coming from the commutation relation of $(\gamma)^p$ with γ^m. By comparing with the d-dimensional result, we get

$$\Delta^{-\frac{1}{d-2}}E_m^\mu V^{m,IJ}U^{I'}{}_I U^{J'}{}_J = \frac{2c}{s}(\Pi^{-1})_i{}^A(\Pi^{-1})_j{}^B V_{AB}^\mu(\gamma^{ij})^{I'J'} \tag{4.16}$$

and therefore we obtain an ansatz for the vielbein,

$$\Delta^{-\frac{1}{d-2}}E_m^\mu = \frac{2ce}{s}(\Pi^{-1})_i{}^A(\Pi^{-1})_j{}^B V_{AB}^\mu Tr(\gamma^{ij}UV_nU^{-1}) \tag{4.17}$$

Here e is defined by $(UV^mU^T)^{IJ}(UV_nU^T)_{IJ} = e\delta_n^m$. Squaring the result for the vielbein we get back the metric in (3.6), provided $4c^2 = s^2 e$.

We also get a self-consistency condition on the matrix U, by plugging (4.17) back into (4.16),

$$e(\Pi^{-1})_i{}^A(\Pi^{-1})_j{}^B V_{AB}^\mu Tr(\gamma^{ij}UV_mU^{-1})V^{m,IJ}U^{I'}{}_I U^{J'}{}_J = (\Pi^{-1})_i{}^A(\Pi^{-1})_j{}^B V_{AB}^\mu(\gamma^{ij})^{I'J'} \tag{4.18}$$

This equation is the most general condition on the matrix U. In [6], we found the equation (in the $AdS_7 \times S_4$ case)

$$U\not{Y} = \not{v}U, \quad v_i = \Delta^{-3/5}(\Pi^{-1})_i{}^A Y_A \tag{4.19}$$

and showed that it implies (4.18). But the reverse is also true: (4.18) implies (4.19). If one multiplies (4.18) by $(\gamma_{kl})_{I'J}$, one gets on the left-hand side:

$$Tr(U\not{Y}U^{-1}\gamma_{kl}U\not{Y}U^{-1}\gamma^i\gamma^j)(\Pi^{-1})_i{}^A Y_A(\Pi^{-1})_j{}^B C^B_\mu \tag{4.20}$$

But $U\not{Y}U^{-1}$ will be equal to $\not{v}$, where v is a unit vector, just because U is a SO(n+1) matrix in the spinor representation. To determine v, we impose that we recover the right-hand side and obtain

$$v_i = \Delta^{-3/5}(\Pi^{-1})_i{}^A Y_A \tag{4.21}$$

as desired. So we can say that (4.18) is the most general equation on U, but it can be brought to the nicer form (4.19).

Although we look here at the case $AdS_7\times S_4$, this is valid for all (d,n), the only thing we used here was that $TrM\gamma_A TrN\gamma_A \propto TrMN$ (A.8), under the assumption $TrM = TrN = 0$, and that $U\not{Y}U^{-1} = \not{v}$ for some v. Then we get in general

$$v_i = \Delta^{\beta n/2-1}(\Pi^{-1})_i{}^A Y_A \tag{4.22}$$

We will now finally turn to the specific cases of the $AdS_4\times S_7$ and the $AdS_5\times S_5$ compactifications.

For the $AdS_4\times S_7$ compactification, the full vielbein ansatz (with no restriction to the Sl(8)/SO(8) scalar coset) was implicitly derived in [2], by the equation

$$\frac{\sqrt{2}}{8}i\Delta^{-1/2}E^\mu_m(UV^mU^T)_{ij} = w_{ij}{}^{IJ}V^\mu_{IJ} \tag{4.23}$$

and we can derive the vielbein and the equation for U from it. The vielbein is

$$\frac{\sqrt{2}}{8}i\Delta^{-1/2}E^\mu_m = -\frac{1}{16}w_{ij}{}^{IJ}V^\mu_{IJ}(UV^mU^T)_{ij} \tag{4.24}$$

(we used here that $(UV^mU^{-1})_{ij}(UV^nU^{-1})^{ij} = -16\delta^n_m$) and the equation for U (the selfconsistency equation) is

$$-\frac{1}{16}w_{kl}{}^{IJ}V^\mu_{IJ}(UV^mU^T)_{kl}(UV^mU^T)_{ij} = w_{ij}{}^{IJ}V^\mu_{IJ} \tag{4.25}$$

We note that now we can't find a nicer form for the U equation, because the vielbein w doesn't factorize into two, as it does if we restrict to the $Sl(8)/SO(8)$ coset. We also note that from the vielbein (4.23) one can rederive the metric (3.31).

For the $AdS_5\times S_5$ case, the ansatz for the vielbein can be obtained easily via the general procedure outlined before. From the 10d transformation law, we get

$$\delta B^\mu_\alpha \frac{1}{2}\Delta^{-1/3}E^\mu_m[\varepsilon\gamma^m\psi_\alpha + \text{spin } 1/2 \text{ terms}] \tag{4.26}$$

whereas the 5d susy law is

$$\delta B^{AB}_\alpha = 2i(\Pi^{-1}(\mathbf{15}))_{ab}{}^{AB}(\bar{\epsilon}^a\psi^b_\mu + \text{spin } 1/2 \text{ terms}) \tag{4.27}$$

implying an equation for the vielbein,

$$\Delta^{-1/3}E^\mu_m V^{m,IJ}U^a{}_I U^b{}_J = 4i(\Pi^{-1}(\mathbf{15}))_{ab}{}^{AB}V^\mu_{AB} \tag{4.28}$$

Here $U^a{}_I$ is a matrix in the coset $USp(8)/SO(6)_g$. Using $(UV^mU^{-1})_{ab}(UV_nU^{-1})^{ab} = -4\delta^n_m$ we can square (4.28) to get back (3.39), as promised. The vielbein then is

$$\Delta^{-1/3}E^\mu_m = -i(\Pi^{-1}(\mathbf{15}))_{ab}{}^{AB}V^\mu_{AB}V^{m,IJ}U^a{}_IU^b{}_J \tag{4.29}$$

and the selfconsistency equation (the equation for the matrix U) is

$$-\frac{1}{4}(\Pi^{-1}(\mathbf{15}))_{ab}{}^{AB}V^\mu_{AB}V^{m,IJ}U^a{}_IU^b{}_JV^{m,IJ}U^a{}_IU^b{}_J = (\Pi^{-1}(\mathbf{15}))_{ab}{}^{AB}V^\mu_{AB} \tag{4.30}$$

A comment is in order for the fermionic ansatz for the $AdS_5 \times S_5$ case: in 10d we already have two gravitinos, which one can take to form a complex gravitino. Then the KK reduction will have the same form as the general one, but now the Killing spinors are in the spinorial representation of $SO(6)_g$, or the fundamental representation of $SU(4)$, which is a 4d complex representation. The 5d gravitinos are in the fundamental of $USp(8)_c$, which is a 8d real representation, so one has a choice of writing the U matrix as an 8 by 8 real matrix, or a 4 by 4 complex one.

5 Conclusions and discussions

We have shown that the ansatz in [6] can be used for a variety of purposes. By further truncating the number of fields, we were able to reproduce the ansätze of [15, 16, 13, 22, 14]. That is to be expected since the KK truncation of 11d sugra on $AdS_7 \times S_4$ in [6] is a *consistent* one (i.e. satisfies the higher dimensional equations of motion and susy laws), as are the truncations found in [15, 16, 13, 22, 14]. As a byproduct of our analysis, we find a lagrangean formulation of the $\mathcal{N} = 2$ model, which was missing in the literature. This $\mathcal{N} = 2$ model was described until now only through equations of motion, supplemented with the self-duality constraint. The lagrangean which describes the bosonic sector of the N=2 model reads:

$$\begin{aligned}\sqrt{g_7^{-1}}L_{7d,\mathcal{N}=2} \quad = \quad & R + g^2(2X^{-3} + 2X^2 - X^{-8}) + 5\partial_\alpha X^{-1}\partial_\alpha X + \frac{g^2}{2}X^{-4}S_{\alpha\beta\gamma}S^{\alpha\beta\gamma} \\ & -\frac{1}{4}X^{-2}F^i_{\alpha\beta}F^{\alpha\beta\, i} + \epsilon^{\alpha\beta\gamma\delta\epsilon\eta\zeta}\sqrt{g_7^{-1}}\left(\frac{g}{24\sqrt{2}}S_{\alpha\beta\gamma}F_{\delta\epsilon\eta\zeta}\right. \\ & \left. + (-\frac{1}{48\sqrt{2}g}\omega_{\alpha\beta\gamma} + \frac{i}{8\sqrt{3}}S_{\alpha\beta\gamma})F^i_{\delta\epsilon}F^i_{\eta\zeta}\right)\end{aligned} \tag{5.1}$$

where the three form $A_{(3)}$ used so far in the $\mathcal{N} = 2$ model is a linear combination of the three form $S_{(3)}$ of the maximal gauged sugra model and the Chern-Simons three form $\omega_{(3)}$ (2.35).

Then we have found that if we make the KK reduction on a sphere S_n of a theory involving gravity and an antisymmetric tensor $F_{(n)}$ to a theory involving gravity and scalars, the scalar coset has a submanifold $Sl(n+1)/SO(n+1)_c$, and the ansatz for the line element is

$$ds^2_D = (Y\cdot T\cdot Y)^{-\frac{2}{(d-2)(2-\beta n)}}ds^2_7 + (Y\cdot T\cdot Y)^{\frac{\beta}{2-\beta n}}T^{-1}_{AB}DY^ADY^B \tag{5.2}$$

where $\beta = \frac{2}{n-1}\frac{d-1}{d-2}$ and $DY^A = dY^A + gB^{AB}Y_B$ is a covariant derivative. For the antisymmetric tensor, the ansatz for the dependence on the coset scalars is

$$\begin{aligned}
A_{(n-1)} &= -\Delta^{3-\beta n-\frac{\alpha}{2}} a\epsilon_{A_1\ldots A_{n+1}} dY^{A_1}\wedge \ldots \wedge dY^{A_{n-1}} Y^{A_n}\left(\frac{T\cdot Y}{Y\cdot T\cdot Y}\right)^{A_{n+1}} + \\
&\quad -\frac{1}{n}\frac{\overset{\circ}{D}_{\mu_n}}{\overset{\circ}{\Box}}(\epsilon_{\mu_1\ldots\mu_n}\sqrt{\overset{\circ}{g}}\Delta^{3-\beta n-\frac{\alpha}{2}}) \qquad (5.3)\\
F_{(n)}|_{no\ \partial T} &= \Delta^{3-\beta n-\frac{\alpha}{2}}(\sqrt{\overset{\circ}{g}}\epsilon_{\mu_1\ldots\mu_n}dx^{\mu_1}\ldots dx^{\mu_n})\,(1+ \\
&\quad +\frac{a}{n}\left[\frac{T}{Y\cdot T\cdot Y}-(n+1)-2\left(\frac{Y\cdot T^2\cdot Y}{(Y\cdot T\cdot Y)^2}-1\right)\right]) \\
&\quad +2a\left(1-\frac{\alpha-2}{2(2-\beta n)}\right)\left(1-\frac{Y\cdot T^2\cdot Y}{(Y\cdot T\cdot Y)^2}\right) \qquad (5.4)
\end{aligned}$$

and we see that by making derivatives covariant we can couple to gauge fields as in the metric. But there are other terms involving the Yang-Mills field strength in the ansatz for the antisymmetric tensor, as we can easily see in the $AdS_7\times S_4$ example. By imposing Bianchi identity on $F_{(n)}$ we can presumably generate these terms. However, it is possible to miss a separately closed form, gauge-invariant, and which vanishes when the scalar fields are set to zero. So, the ansatz for the antisymmetric tensor field strength generated by this method should be checked in the action too, to verify its completeness.

Moreover, we note that the ∂T terms in $F_{(n)}$ contain $1/\Box_x$ terms, and so we were not able to reproduce the scalar field kinetic terms in the d dimensional action, except in the cases of $AdS_4\times S_7$, $AdS_7\times S_4$ and $AdS_5\times S_5$, when these $1/\Box_x$ terms disappear already in $F_{(n)}$.

In the $AdS_5\times S_5$ case, we got a line element,

$$ds_{10}^2 = \Delta^{-2/3}(ds_5^2 + N_{AB}DY^A DY^B) \qquad (5.5)$$

where N_{AB} is the 'inverse' of $\tilde{N}_{AB} = (\Pi^{-1}(\mathbf{15}))_{ab}{}^{AC}Y_C(\Pi^{-1}(\mathbf{15}))_{ab}{}^{BD}Y_D$, i.e.

$$\tilde{N}^{AC}N_{BC} = \delta^A_D + (Y_D\text{terms}) \qquad (5.6)$$

One can now use this to lift any solution of gauged sugra to 11 dimensions, at least for the metric.

We were able to obtain the vielbein too, by first fixing a small part of the fermion ansatz, namely the relation between the D dimensional and the d dimensional gravitinos. We found that also in the general case we need a matrix $U^{I'}{}_I$ interpolating between the composite symmetry group indices of the gravitino and the gauge group spinor indices of the Killing spinors. This matrix apppears in the ansatz for the vielbein, which for the case of the $Sl(n+1)/SO(n+1)$ coset reads

$$\Delta^{-\frac{1}{d-2}}E^\mu_m = \frac{2ce}{s}(\Pi^{-1})_i{}^A(\Pi^{-1})_j{}^B V^\mu_{AB}Tr(\gamma^{ij}UV_nU^{-1}) \qquad (5.7)$$

In this case, the matrix U obeys the nice equation

$$U\not{Y} = \not{v}U,\ \ v_i = \Delta^{\beta n/2-1}(\Pi^{-1})_i{}^A Y_A \qquad (5.8)$$

which probably has the most general solution very similar to the one found in [6] for the $AdS_7 \times S_4$ case, but we did not investigate that further.

We have also found the full ansatz for the vielbein in the $AdS_5 \times S_5$ case, as well as the U equation. While these probably have less practical applications, they will be useful if one wants to check the consistency of the $AdS_5 \times S_5$ KK reduction, which is the one thing notably missing in the bussiness of consistent truncations (it's the case most used for the AdS-CFT correspondence). We note that if one does not restrict to the $Sl(6)/SO(6)$ scalar coset, the equation for the matrix U does not take a nice form, but instead is found in (4.30).

Note added After the first version of the paper was posted, we became aware of the paper [37], where the form of the 5d metric (5.5) was conjectured.

Acknowledgements We are grateful to Peter van Nieuwenhuizen for discussions and for the the collaboration to an earlier paper [6], from where most of the work reported here originated. We are also grateful to Chris Pope for a discussion of his work, which led us to the analysis in section 2, and to Radu Roiban for discussions of our results.

Appendix A

A.1 Killing spinors on spheres

A Killing spinor $\eta(x)$ on a sphere S_n parametrized by the coordinates x^μ is defined by:

$$D_\mu \eta(x) = cm\gamma_\mu \eta(x) \tag{A.1}$$

where we introduced a dimensionful constant m, and c is a another dimensionles constant which will be fixed soon. Spheres are Einstein spaces, and moreover, are maximally symmetric

$$R_{\mu\nu}{}^{mn} = m^2(e^m_\mu(x)\, e^n_\nu(x) - e^n_\mu(x)\, e^m_\nu(x)) \tag{A.2}$$

Then, the integrability condition reads

$$[D_\mu, D_\nu]\eta = \frac{1}{4}R_{\mu\nu}{}^{mn}\gamma_{mn}\eta = \frac{m^2}{2}\gamma_{\mu\nu}\eta = -2c^2m^2\gamma_{\mu\nu}\eta \tag{A.3}$$

and we derive that $c = \pm i/2$[¶]. Corresponding to the sign of c we define two sets of Killing spinors, $D_\mu\eta_\pm = \pm 1/2m\gamma_\mu\eta_\pm$. Spheres are the only manifold where we can define both sets $\eta_\pm$. An explicit representation can be given in terms of stereographic cordinates (see [36]):

$$e^m_\mu(z) \;=\; \frac{\delta^m_\mu}{1+z^2} \qquad \omega^{mn}_\mu(z) = \frac{-2\delta^m_\mu z^n + 2\delta^n_\mu z^m}{1+z^2} \tag{A.4}$$

$$\eta_\pm(z) \;=\; \frac{1 \pm i\gamma_m \delta^m_\mu z^\mu}{\sqrt{1+z^2}}\eta_\pm(0) \tag{A.5}$$

We define further

$$\bar\eta \equiv \eta^T C \tag{A.6}$$

[¶]In general, using the integrability condition we derive that Killing spinors exist only on Einstein spaces, namely $R_{\mu\nu} = 4c^2(n-1)g_{\mu\nu}$.

where C is the charge conjugation matrix: $\gamma_\mu^T = \lambda C\gamma_\mu C^{-1}$ and $C^T = \epsilon C$.

S_n	ϵ	λ	$\bar\eta^I\eta^J$	*gauge group*	*number of* $\eta_+{}'s$
$n=2$	$\pm$	$\pm$	ϵ^{IJ}	$SO(3)\simeq SU(2)$	2
$n=3$	$-$	$-$	ϵ^{IJ}	$SO(4)\simeq SU(2)\times SU(2)$	4
$n=4$	$-$	$\pm$	Ω^{IJ}	$SO(5)\simeq USp(4)$	4
$n=5$	$-$	$+$	Ω^{IJ}	$SO(6)\simeq SU(4)$	8
$n=6$	$\pm$	$\pm$	δ^{IJ}	$SO(7)$	8
$n=7$	$+$	$-$	δ^{IJ}	$SO(8)$	8
$n=8$	$+$	$\pm$	δ^{IJ}	$SO(9)$	8
$n=9$	$+$	$+$	δ^{IJ}	$SO(10)$	16

The labeling index I is in the spinor representation of the gauge group.

For example, if $\gamma_\mu^T = -C\gamma_\mu C^{-1}$, then $\bar\eta^I_\pm(z)\eta^J_\pm(z) = \bar\eta^J_\pm(0)\eta^I_\pm(0)$ and the Killing spinors are normalized to either Ω^{IJ} or δ^{IJ} (if $C_{n+1} = C_{(-)}$, or $C_{(+)}$ respectively) depending on the sign of ϵ: $\bar\eta^I\eta^J = (\bar\eta^I\eta^J)^T = \epsilon\bar\eta^I\eta^J$. In this case, we can express the Cartesian coordinates Y^A parametrizing the sphere (with Y^A in the vector representation of the gauge group) as:

$$Y^A = \Gamma^A_{IJ}\bar\eta^I_+\eta^J_- \tag{A.7}$$

where in even dimensions $\Gamma^A = \{i\gamma_m\gamma_5, \gamma_5\}$, and in odd dimensions we use the Γ^A in the chiral representation of $SO(n+1)$. Substituting (A.5) into (A.7) we can check that with an appropriate choice of $\eta_+(0)$ and $\eta_-(0)$ we indeed find $Y^AY^A = 1$.

A similar analysis can be done for the case $\lambda = +1$.

Another useful relation is the completeness relation of the Clifford algebra matrices:

$$\gamma^\mu{}_{\alpha\beta}\gamma^\mu{}_{\gamma\delta} = 2^{[n/2]-1}\lambda(C_{\alpha\gamma}C_{\beta\delta} + \lambda\epsilon C_{\beta\gamma}C_{\alpha\delta}) + A(\lambda+1)\epsilon C_{\alpha\beta}C_{\gamma\delta} \tag{A.8}$$

where $2A = \epsilon n - 2^{[n/2]-1}\lambda(1 + 2^{[n/2]}\lambda\epsilon)$.

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